



2018 State STEM Competition

High School Math Test

Student Instructions

- This test is for students in 9th, 10th, 11th and 12th grades. If you are not in one of those grades, work with a test proctor to find the appropriate test.
- Leave the test booklet closed and wait until the proctor tells you to start.
- The test period will be 60 minutes and this test contains 60 multiple choice questions.
- This test includes topics covered in all high school math classes. The questions range in difficulty from easy to very challenging. Most students will NOT be familiar with all of the material on their test.
- There is no penalty for skipping a problem or answering incorrectly.
- You may write on this test. If you need scratch paper, please raise your hand and ask for some.
- Your score will be determined by the number of correct answers. All ties will be broken by awarding the place to the student having the most consecutive correct answers from the start of the test.
- You will not be permitted to leave the room while the test is in progress. If you finish early, you must remain in the room quietly until the test is completed. Any student causing a disturbance will be disqualified.
- **LEGIBLY** write your name, grade, and school on the right side of the answer/Scantron sheet, as shown below -

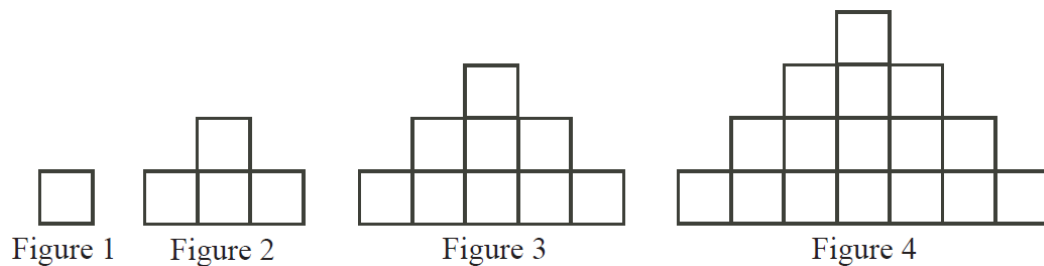
NAME	Your First and Last Name		
SUBJECT	Your Grade	TEST NO.	Test Name
DATE	School Name	PERIOD	

- Answers must be marked on the answer/Scantron sheet with a number two pencil.
- All tests must be turned in at the end of the testing period. Tests may not be taken from the testing room.

GOOD LUCK!

Answer the following questions by marking the **BEST** answer on the answer sheet.

- (1) Z varies inversely with A^2 and $Z = 4$ when $A = 3$. Find A when $Z = 9$.
 A) 1 B) 2 C) 3 D) 4
- (2) Albert weighs 88 pounds and is sitting on a seesaw, $5\frac{1}{2}$ feet from the middle. Liz weighs 66 pounds and is sitting 6 feet from the center and on the other side. How much additional weight does Liz need to balance the seesaw?
 A) $15\frac{2}{3}$ lbs B) $15\frac{1}{3}$ lbs C) 15 lbs D) $14\frac{2}{3}$ lbs
- (3) If the determinant of $\begin{bmatrix} 8 & 2 \\ 3 & x \end{bmatrix} = 6x$, then what is the value of x ?
 A) 4 B) 3 C) 2 D) 1
- (4) What type of graph is produced by the equation $2x^2 - 2y^2 - 4x + 6y = 10$?
 A) parabola B) ellipse C) hyperbola D) lemniscate
- (5) If $y = -x^2 + 16$, where $y > 0$, and x is an integer, then how many distinct solutions exist?
 A) 7 B) 6 C) 8 D) 9
- (6) Consider the following sequence of figures showing arrangements of square tiles in the figures below. More figures can be drawn, each having one row of tiles more than the previous figure. This new bottom row is constructed using two tiles more than the number of tiles in the bottom row of the previous figure. Determine the positive difference between the total number of tiles in Figure 11 and the total number of tiles in Figure 9.

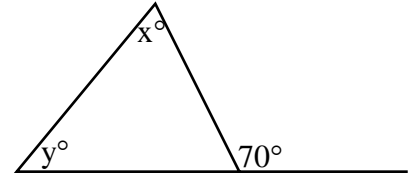


- A) 19 B) 21 C) 36 D) 40
- (7) The average of 6, 9 and 18 is equal to the average of 12 and y . What is the value of y ?
 A) 22 B) 21 C) 10 D) 11
- (8) Deb can retire when her age and the number of years that she has worked add to 85. At present, she is 50 years old and has worked for 19 years. If she works continuously until she retires, how old will she be when she can retire?
 A) 54 B) 58 C) 66 D) 69
- (9) Find the perimeter of a regular polygon with sides of length 15-inch and interior angles measuring 160° each.
 A) 220 in. B) 240 in. C) 270 in. D) 300 in.

- (10) Find the equation of the line that is a perpendicular bisector of the line segment with end points (8, 14) and (2, 6).
 A) $3x + 4y = 55$ B) $4x - 3y = -10$ C) $4x + 3y = 50$ D) $3x - 4y = -25$

- (11) In the figure to the right, determine the value of: $m\angle x^\circ + m\angle y^\circ$.

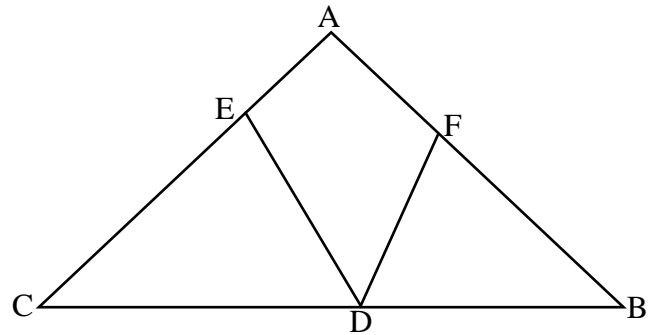
- A) 20°
 B) 35°
 C) 70°
 D) 140°



- (12) The interior angles of a polygon are in arithmetic progression. The least angle is 120° and common difference is 5° . Find the least number of sides that this polygon can have.
 A) 7 B) 9 C) 12 D) 16

- (13) Using the figure to the right: in triangle ABC , $AB = AC$. The measure of angle A is 80° . If $CE = CD$ and $BF = BD$, determine the measure of angle EDF . (drawing not to scale)

- A) 25°
 B) 30°
 C) 40°
 D) 50°

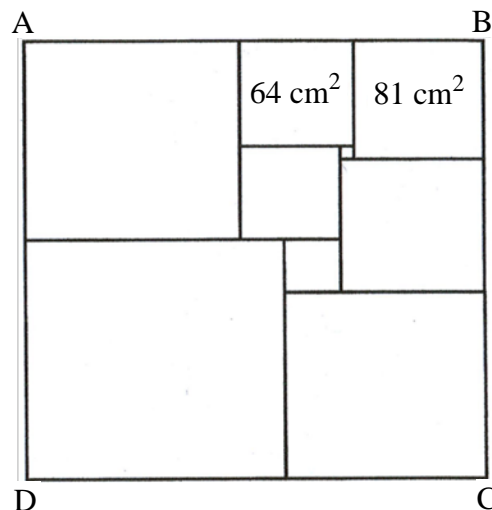


- (14) A convex polyhedron has 64 edges and 50 faces. How many vertices does it have?
 A) 16 B) 18 C) 22 D) 24
- (15) The median of a data set with 51 observations is 23.5. If the largest observation is increased by a value of 0.5, what will the new median be?
 A) 23 B) 23.5 C) 24 D) 24.5
- (16) Ronan has 8 clean shirts, 6 clean pairs of pants, and 10 clean pairs of socks. It's cold outside, so Ronan wants to wear two of the clean shirts, one pair of pants, and one clean pair of socks. Assuming Ronan does not mix socks from different pairs, in how many ways can he select his outfit from those articles of clothes? Note that an outfit is the same regardless of the way in which it gets worn.
 A) 840 B) 1680 C) 1920 D) 3360
- (17) The letters in the word TURTLE are written on tiles, one letter per tile. The six tiles are then placed in a bag. Two letters are drawn back-to-back at random and without replacement. Find the probability that the letter on the first tile is a consonant and the letter on the second tile is a vowel.
 A) $\frac{2}{9}$ B) $\frac{4}{15}$ C) $\frac{4}{9}$ D) $\frac{8}{15}$
- (18) If $x < 2$, then $|x - 2|$ equals
 A) $x - 2$ B) $x + 2$ C) $2 - x$ D) $-2 - x$

- (19) A certain company determines its monthly revenue function to be $R(x) = -3x^2 + 96x + 872$, where x is the number of items manufactured. How many items need to be manufactured to maximize revenue?
A) 16 B) 32 C) 1536 D) 1640
- (20) What is the value of $i^{1775} + i^{1812} + i^{1861} + i^{1941}$?
A) 0 B) 1 C) $1 - i$ D) $1 + i$
- (21) How many zeroes does $50!$ (50 factorial) end with?
A) 5 B) 10 C) 12 D) more than 16
- (22) Liz is eager to get to the city of Orange and drives at a rate of 80 mph going there. After having a great time, she is sad to leave and drives home at a rate of 48 mph. Find her average rate for the entire trip.
A) 66 mph B) 64 mph C) 62 mph D) 60 mph
- (23) If Matt scores 99% on his next math test, his average will be 91%. If he gets 81% on his next test, his average will be 89%. How many tests has Matt already taken?
A) 6 B) 7 C) 8 D) 9
- (24) Bacteria in an enclosed petri dish multiply at an exponential rate according to the equation $P = P_0e^{ct}$, where P_0 is the original population, t is time in hours, and c is a constant. If 2000 bacteria are placed in a petri dish at 10:00 AM and at 11:00 AM there were 4000 bacteria there, then how many will be in the dish at 1:00 PM?
A) 2,000 B) 8,000 C) 16,000 D) 128,000
- (25) What is true about the following system of equations: $\begin{cases} 3x - 2y = 1 \\ -6x + 4y = -2 \end{cases}$?
A) This system of equations is both dependent and consistent.
B) This system of equations is both independent and consistent.
C) This system of equations is both dependent and inconsistent.
D) This system of equations is both independent and inconsistent.
- (26) Solution A is 12% acid and solution B is 60% acid. How much of solution A should be mixed with solution B to get 24 liters of a solution that is 50% acid?
A) 5 liters B) 6 liters C) 18 liters D) 19 liters
- (27) A club of 15 members must choose a delegation of 4 members to serve at a convention. Ruby and Ryanna each refuse to serve in the delegation unless the other one also serves. How many delegations are possible?
A) 793 B) 806 C) 819 D) 832
- (28) What are the coordinates of the point on the line $y = 2x + 1$ that is closest to the point $(4, 2)$?
A) $\left(\frac{8}{5}, \frac{21}{5}\right)$ B) $\left(\frac{6}{5}, \frac{17}{5}\right)$ C) $\left(-\frac{2}{3}, -\frac{1}{3}\right)$ D) $\left(\frac{4}{3}, \frac{11}{3}\right)$
- (29) What is the units digit for $1! + 2! + 3! + 4! + \dots + 1000!$?
A) 1 B) 3 C) 7 D) 9

- (30) Genny ordered a box of dishes from a website. When they arrived, she found only two of the dishes were undamaged. In fact, two-thirds of the dishes were cracked, one-half of them were chipped, and one-fourth of them were both cracked and chipped! If we let n be the number of dishes she ordered, which of the following is true of n ?
 A) n is a multiple of 6 B) n is a multiple of 5 C) n is a prime number D) n is a perfect square
- (31) Mike sketched the graph of the function $f(x)$ and his wife Paige sketched the graph of $g(x) = 1 - f(x - 2)$. Which of these listed below would describe what you could do to Mike's graph to turn it into Paige's? (Note: These answers describe the steps in order from first step to last step, and the word "flip" refers to reflecting the graph vertically across the x -axis.)
 A) flip, shift up 1, then shift left 2
 B) shift up 1, flip, then shift left 2
 C) shift up 1, flip, then shift right 2
 D) shift down 1, flip, then shift right 2
- (32) Which of the following sequences is not geometric?
 A) 27, 9, 3, 1, ... B) 5, -15, 45, -135, ... C) x, x^3, x^5, x^7, \dots D) 1, 4, 9, 16, ...

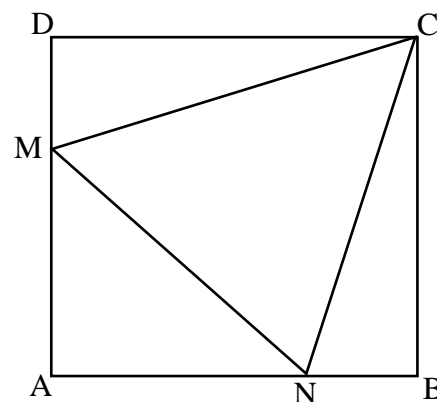
- (33) Quadrilateral $ABCD$ has been dissected into squares – as shown to the right. The area of the square in the upper right corner is 81 cm^2 units. The area of the square to the left of that square is 64 cm^2 . What is the area of the quadrilateral $ABCD$?



- A) 1024 cm^2
 B) 1049 cm^2
 C) 1055 cm^2
 D) 1056 cm^2

- (34) What is the next term in the sequence 5, 6, 14, 32, 64, 115, 191, ...?
 A) 299 B) 283 C) 267 D) 244

- (35) For the figure to the right $ABCD$ is a square and triangle CMN is an equilateral triangle. If the area of the square $ABCD$ is 1, then What is the area of triangle CMN ?

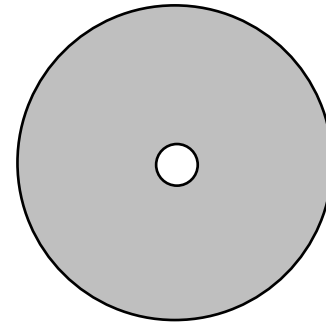


- A) $2\sqrt{3} - 3$
 B) $\frac{2\sqrt{3}}{9}$
 C) $\frac{\sqrt{3}}{4}$
 D) $\frac{\sqrt{2}}{3}$

- (36) A triangle with sides of length 10, 12 and 14 is similar to a triangle whose longest side is 7. Find the area of the smaller triangle.

A) $4\sqrt{6}$ B) $6\sqrt{6}$ C) $9\sqrt{6}$ D) $36\sqrt{6}$

- (37) Two concentric circles form the given annulus (the shaded part in the figure to the right). If the area of the annulus is 825π and the diameter of the smaller circle is 8, find the diameter of the larger circle.



A) 54
B) 56
C) 58
D) 60

- (38) Which line segments of a triangle are concurrent at a point which is equidistant from the three vertices of the triangle?

A) the perpendicular bisectors
B) the medians
C) the angle bisectors
D) the altitudes

- (39) Evaluate $\lim_{t \rightarrow 0} \frac{(5 \sin(4t))}{3t}$.

A) 0 B) 1 C) $\frac{4}{3}$ D) $\frac{20}{3}$

- (40) What is the slope of the line tangent to the curve $x^3 + y^3 - 9xy = 0$ at the point (2, 4)?

A) 0 B) 1 C) $\frac{4}{5}$ D) $-\frac{4}{3}$

- (41) The function, $g(x) = \frac{(x-3)^2}{1+x^2}$, intersects its horizontal asymptote when

A) $x = \frac{3}{4}$. B) $x = 1$. C) $x = \frac{4}{3}$. D) $x = 3$.

- (42) The volume of a spherical balloon is decreasing at the instantaneous rate of $40 \text{ cm}^3/\text{sec}$, at the moment when its radius is 10 cm. At that moment, how rapidly is its radius decreasing?

A) $\frac{1}{10\pi} \text{ cm/sec}$ B) $\frac{1}{20\pi} \text{ cm/sec}$ C) $10\pi \text{ cm/sec}$ D) $20\pi \text{ cm/sec}$

- (43) What is the solution set of the inequality $5 - |x + 4| \leq -3$?

A) $-2 \leq x \leq 6$ B) $-12 \leq x \leq 4$ C) $x \leq -2$ or $x \geq 6$ D) $x \leq -12$ or $x \geq 4$

- (44) Mike has a total of 100 dimes and quarters. If the total value of the coins is \$14.05, how many quarters does he have?

A) 27 B) 40 C) 56 D) 73

- (45) If f is a linear function, $f(3) = 14$ and $f(-6) = -16$, then $f(0) =$ what value?
A) -10 B) 0 C) 4 D) 12
- (46) Find the area bounded by the graphs of $y = x^3$ and $y = \sqrt[3]{x}$.
A) 1 B) $\frac{3}{4}$ C) $\frac{1}{2}$ D) $\frac{2}{3}$
- (47) A ball is thrown down from the top of a 144-foot tall building with initial velocity -28 ft/sec. Assuming the rate of gravitational acceleration to be -32 ft/sec², find the mean height above the base of the building of the ball during its fall to the ground.
A) 72 ft. B) 85.5 ft. C) 96 ft. D) 101.5 ft.
- (48) A fair coin will be tossed four times. What is the probability that at least one of the four coin tosses will result in a head?
A) $\frac{3}{8}$ B) $\frac{1}{4}$ C) $\frac{1}{2}$ D) $\frac{15}{16}$
- (49) From the top of building A, the angle of elevation to the top of building B, 100 feet away is 38° . The angle of depression to the foot of building B is 50° . How tall, to the nearest foot, is building B?
A) 325 ft. B) 250 ft. C) 200 ft. D) 175 ft.
- (50) For which real values of x is $x^8 - 8x^4 + 16 = 0$?
A) $\{2\}$ B) $\{\sqrt{2}\}$ C) $\{-2, 2\}$ D) $\{-\sqrt{2}, \sqrt{2}\}$
- (51) Let us define a sequence a_n such that the first term of the sequence, a_1 , is 1 and the rest of the sequence is generated using the rule: $a_n = 10^{2-2n}$ for $n \geq 2$. If the first three terms are: 1, 0.01 and 0.0001, what is the sum of all the terms of this sequence?
A) ∞ B) $\frac{100}{99}$ C) $\frac{101}{90}$ D) $\frac{10}{9}$
- (52) If A is a 3×3 matrix and determinant of $A = 3$, then determinant of $(3A)$ equals what?
A) 3 B) 9 C) 27 D) 81
- (53) Find the number of positive integer divisors of 756.
A) 16 B) 18 C) 24 D) 31
- (54) What is the solution to the equation $\log_2(x) + \log_4(x) = 0$?
A) 0 B) 1 C) $\frac{1}{2}$ D) 2
- (55) At any time $t \geq 0$, in days, the rate of growth of a bacteria population is given by $y' = ky$, where y is the number of bacteria present and k is a constant. The initial population is 1,500 and the population quadrupled during the first 2 days. By what factor will the population have increased during the first 3 days?
A) 4 B) 5 C) 6 D) 8

(56) If $x = \frac{1}{3}t$ and $y = 5t + 2$ determine the function $y(x)$.

- A) $y = \frac{5}{3}x + 2$ B) $y = \frac{5}{3}x^2 + \frac{5}{3}x$ C) $y = \frac{5}{3}x + \frac{2}{3}$ D) $y = 15x + 2$

(57) What is the remainder when $x^{10} - 1$ is divided by $x^2 - a$?

- A) $\frac{1}{a}$ B) $a - 1$ C) $a^5 - 1$ D) $a^{10} - 1$

(58) The square to the right is a magic square in that all rows, columns, and main diagonals add up to $3a$. What is the value of the center square?

- A) a
B) $a - 1$
C) $a + 1$
D) $a - 2$

	$a + 5$	$a - 1$
	?	$a - 3$

(59) Where are the inflection points for the following function: $f(x) = \begin{cases} x\sqrt{3-x} & \text{for } 0 \leq x \leq 3 \\ -x\sqrt{3-x} & \text{for } x < 0 \end{cases}$?

- (I) (0, 0) (II) (2, 2) (III) (3, 0)
- A) I only B) I, II, III C) I, II D) I, III

(60) A rectangular box with a square base and no top is to be made from 20 square feet of cardboard. What is the maximum volume of such a box?

- A) $\frac{80\sqrt{5}}{27} \text{ ft}^3$ B) $\frac{20\sqrt{15}}{9} \text{ ft}^3$ C) $\frac{10\sqrt{30}}{9} \text{ ft}^3$ D) 8 ft^3